

Image Processing

- Obtaining the Image
- Analyzing the Image

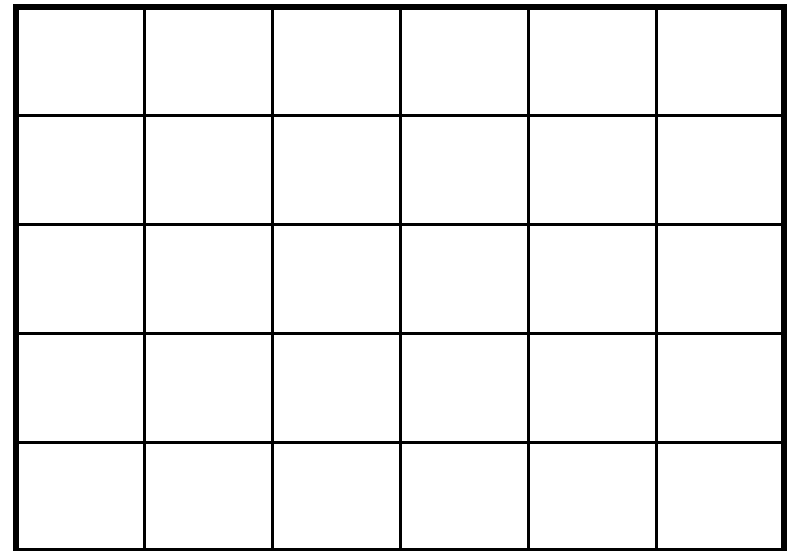
Obtaining the Image

Tomography

- Tomography comes from the Greek root ‘tomos’ which means ‘cut’ or ‘section’
- It is a process of imaging the internal structures of the body by measuring energy which is passed through the body and using a computer to “calculate” an image
 - Energy is passed into the body in a plane perpendicular to the measured structure at various positions in this plane; usually in circular patterns.
 - The energy received by the detector at each position is stored in a computer
 - Using an algorithm, the computer identifies the internal structures within this plane.
 - This is repeated at the next plane until the entire structure is imaged.

Digitization of the Image

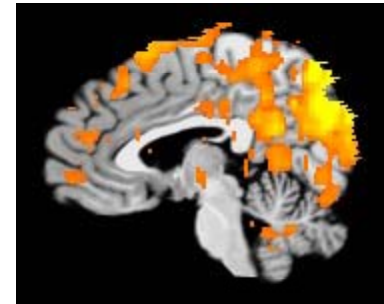
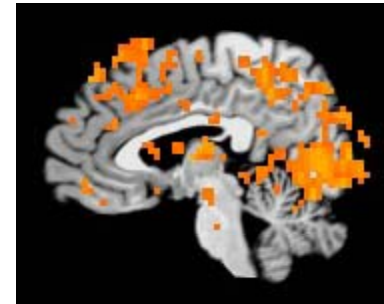
- A 2-D image is divided up into Pixel (picture elements)
- In this case we have $6 \times 5 = 30$ pixels
- Normally, 256×256 , 512×512 , 1024×1024 , etc.
- Each pixel, the intensity of the image is shown as a gray level.
- So let's assume that there are 256 gray levels (white = 0 and black = 255)
 - 8 bits/pixel to distinguish the 256 levels
 - This yields $1024 \times 1024 \times 8$ bits or 1Mbyte
 - Storage and transmission of these files need to be considered
- 3-D is similar except volume elements are used: Voxels



Example of a Digitized Image



http://www.radiologyinfo.org/en/photocat/gallery3.cfm?image=00095488_brain_mets_AP.jpg

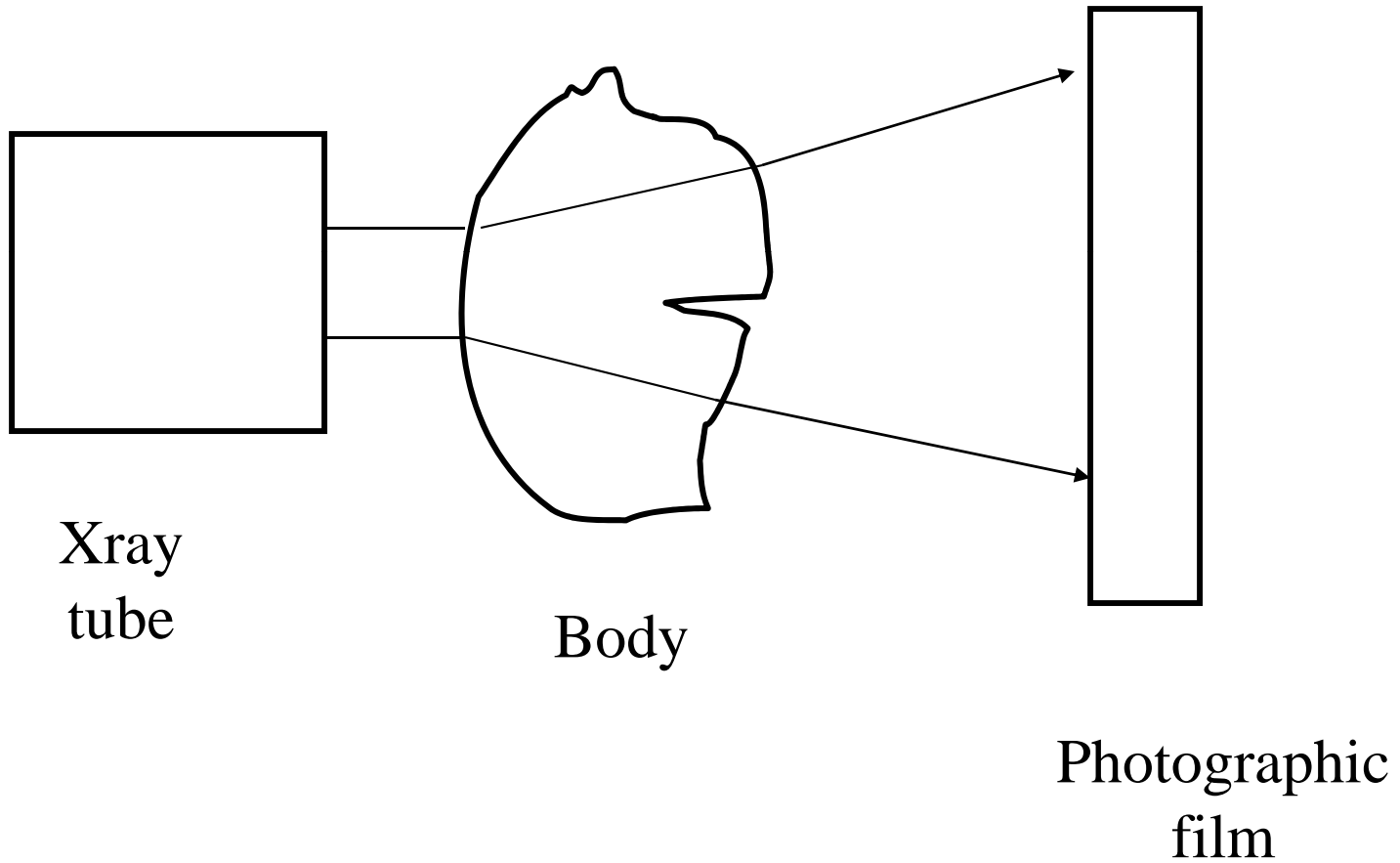


MRI images from Dr. Alvarez

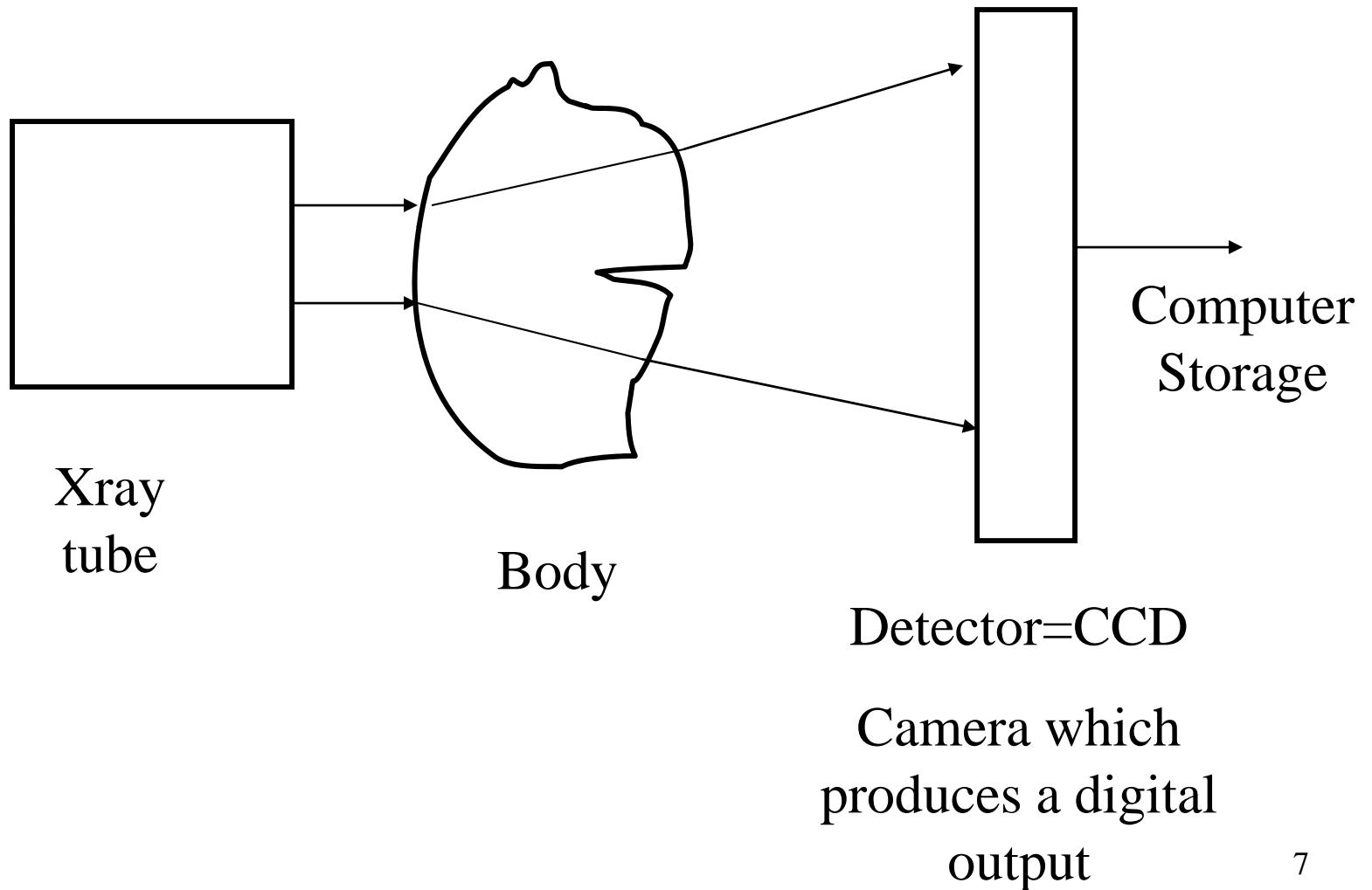
Xray

- As an example of Tomography, let's use Xray energy
- Xrays are electromagnetic, radiation (like light energy) photons having energy of 20 – 150 Kev which is about 10^4 times more energy than light photons
- Xray – discovered in 1895 – oldest form of imaging system.
- Xrays are dangerous because they can dissociate molecules
 - Damage is related to amount and rate of radiation

Older-type Xray System

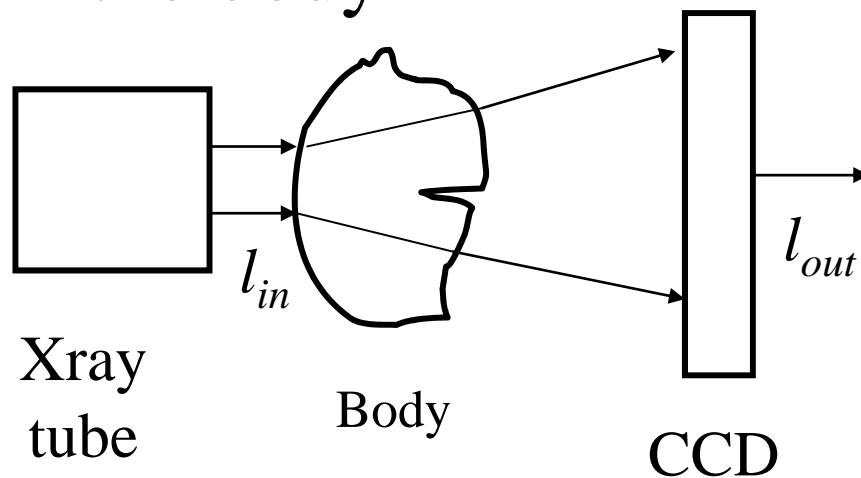


Newer-type Xray System



Propagation of Xrays in the body

- Xrays are attenuated as they move through the body



$$l_{out} = l_{in} e^{-\mu x}$$

l_{out} = final beam intensity

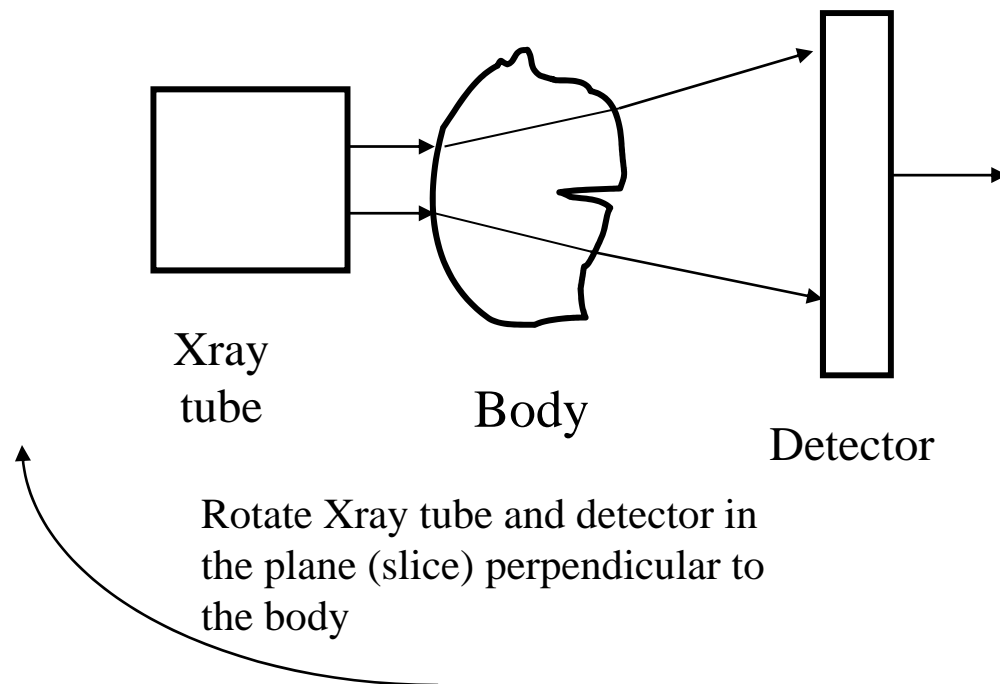
l_{in} = initial beam intensity

μ = attenuation coefficient

x = thickness of the layer of tissue

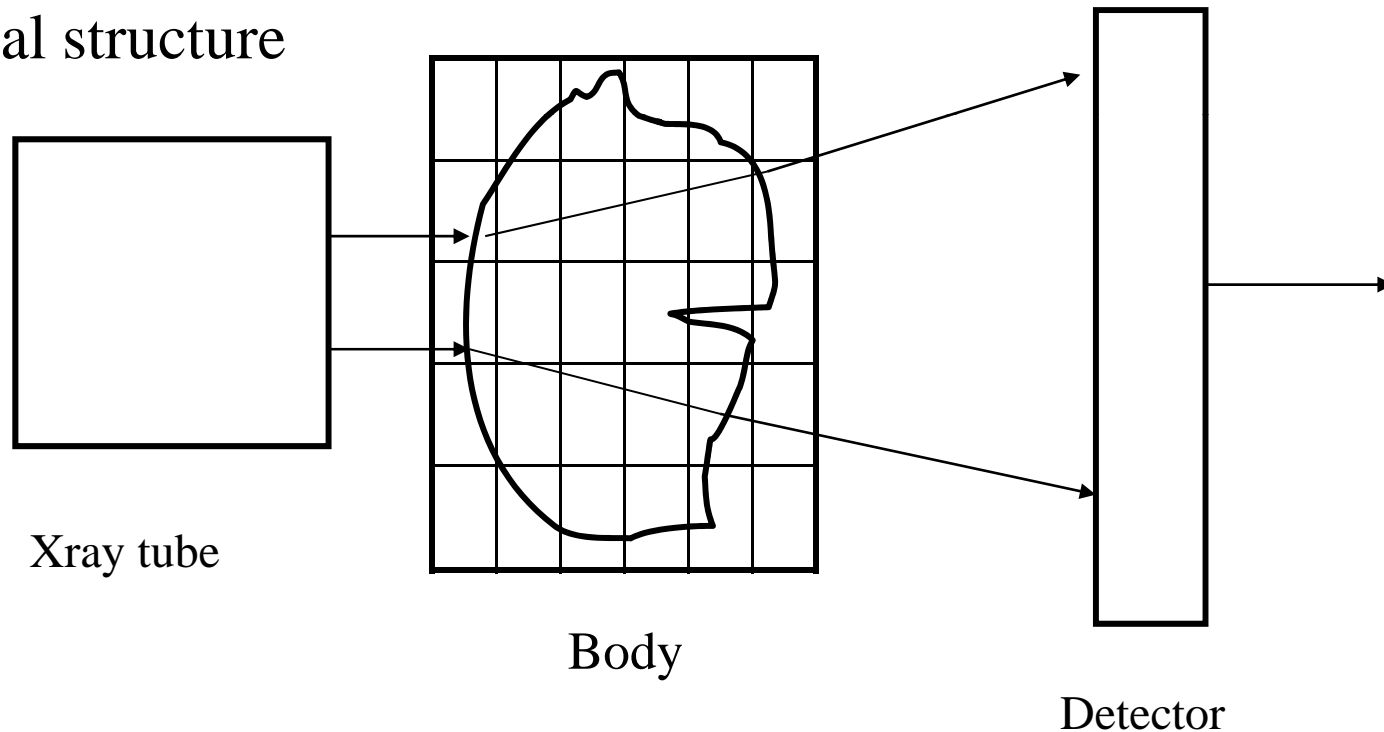
Reconstruction of image

- How do we reconstruct a digital image of the internal structure of the body?



Reconstruction of image

- Divide the plane of the structure into pixels
- For each pixel, there is an “unknown” intensity due to the internal structure



Reconstruction of image

- As an example, a beam passes through the plane and intersects several of the pixels.
- If we assign an intensity g_{ij} to each pixel, we can write an equation relating l_{out} to l_{in} and the g_{ij} 's which the beam passes through.

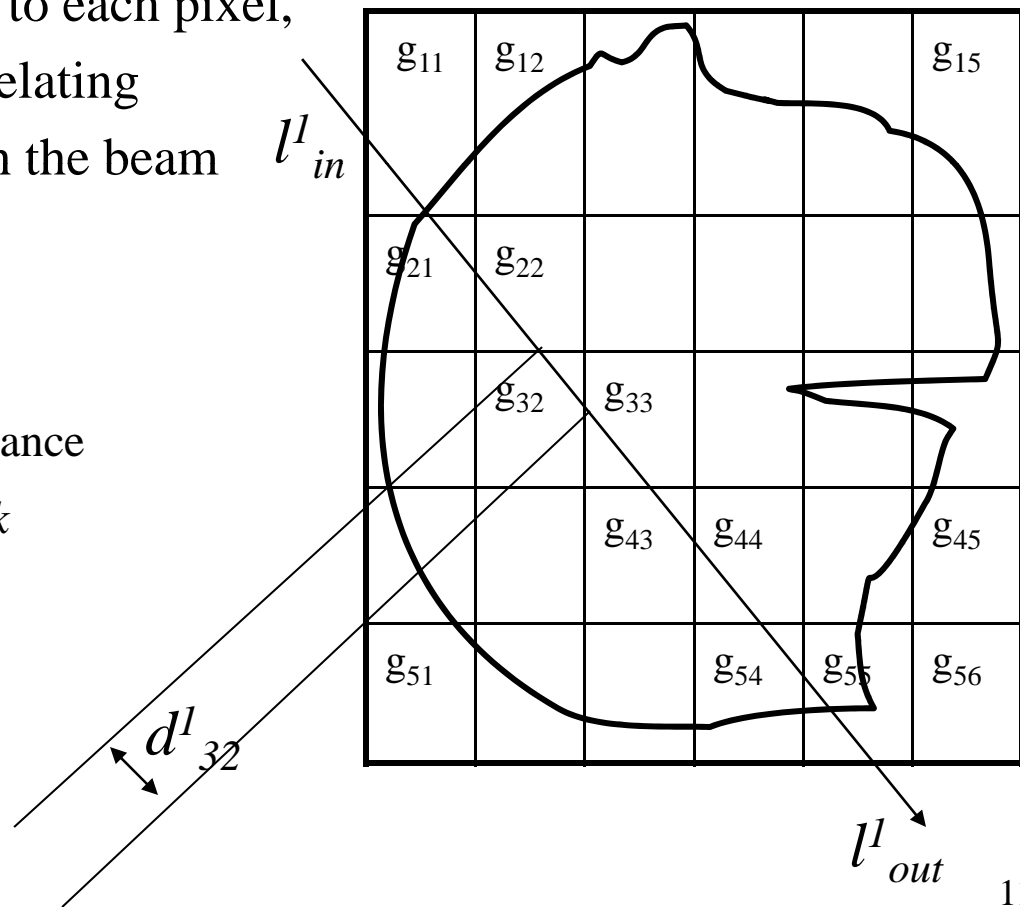
$$l_{out}^1 = l_{in}^1 e^{-g_{11}d_{11}^1} e^{-g_{21}d_{21}^1} \dots e^{-g_{55}d_{55}^1}$$

where the d_{ij}^k 's are the path distance through pixel ij for the beam k

Or

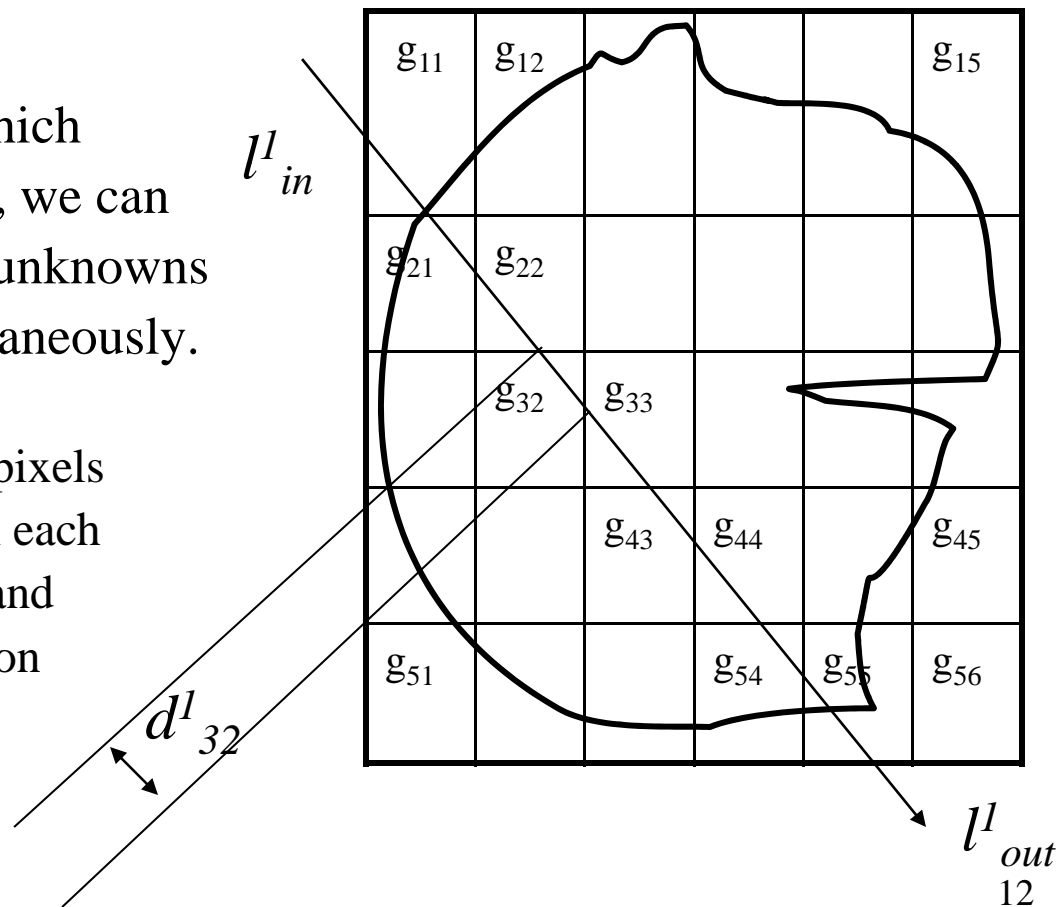
$$\frac{l_{out}^1}{l_{in}^1} = e^{-g_{11}d_{11}^1} e^{-g_{21}d_{21}^1} \dots e^{-g_{55}d_{55}^1}$$

$$\ln \frac{l_{out}^1}{l_{in}^1} = - \sum_{path} g_{ij} d_{ij}^k$$



Reconstruction of image

- If we have $6 \times 5 = 30$ pixels, we can pass 30 beams through this structure and yield 30 simultaneous equations.
- If chose 30 unique paths which encompass all of the pixels, we can have 30 equations with 30 unknowns which can be solved simultaneously.
 - each path passes through a different set of pixels
 - The path distance through each pixel are pre-determined and planned during a calibration run.



Reconstruction of image

- Let's write the equations for the following 2x2 example:

$$\ln \frac{l_o^1}{l_i^1} = -(g_{21}d^1_{21} + g_{11}d^1_{11} + g_{12}d^1_{12}) \quad \text{Beam}_2$$

$$\ln \frac{l_o^2}{l_i^2} = -(g_{11}d^2_{11} + g_{12}d^2_{12} + g_{22}d^2_{22})$$

$$\ln \frac{l_o^3}{l_i^3} = -(g_{12}d^3_{12} + g_{22}d^3_{22} + g_{21}d^3_{21})$$

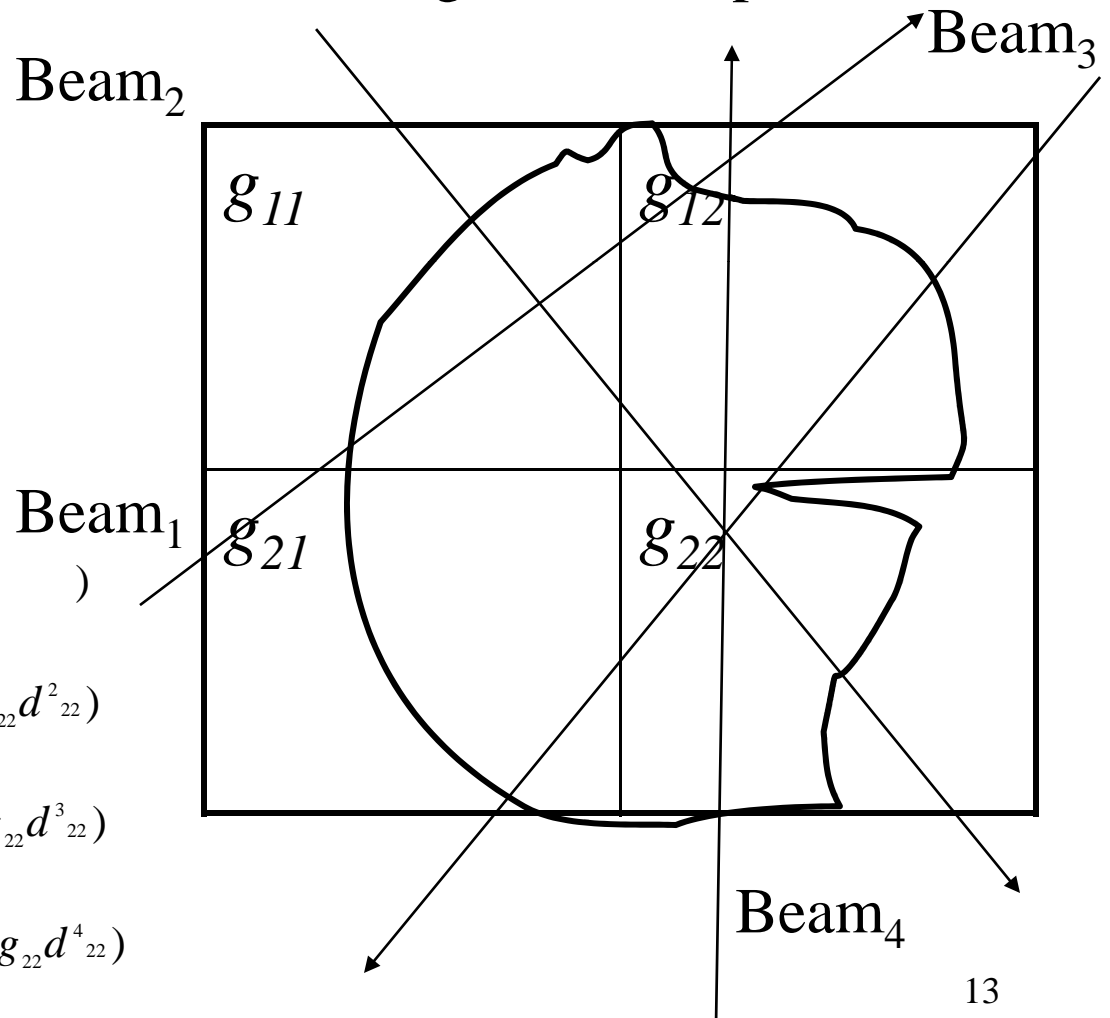
$$\ln \frac{l_o^4}{l_i^4} = -(g_{22}d^4_{22} + g_{12}d^4_{12})$$

$$\ln \frac{l_o^1}{l_i^1} = -(g_{11}d^1_{11} + g_{12}d^1_{12} + g_{21}d^1_{21} + \quad)$$

$$\ln \frac{l_o^2}{l_i^2} = -(g_{11}d^2_{11} + g_{12}d^2_{12} + \quad + g_{22}d^2_{22})$$

$$\ln \frac{l_o^3}{l_i^3} = -(\quad + g_{12}d^3_{12} + g_{21}d^3_{21} + g_{22}d^3_{22})$$

$$\ln \frac{l_o^4}{l_i^4} = -(\quad + g_{12}d^4_{12} + \quad + g_{22}d^4_{22})$$



Reconstruction of the Image

- Once knowing the g_{ij} 's, a image can be displayed which represents the structure of the slice.
- This is repeated for each slice needed to characterize the structure needing imagery.
- We can easily solve the 4 sets of equations to determine the g_{ij} 's using matrix mathematics.
- However, a 4x4 pixel image will clearly provide a poor image.
- So for a higher resolution picture of 1024x1024 pixels, there are 1,048,576 unknowns and, therefore, 1,048,576 beams paths to generate and 1,048,576 equations to solve.
- A computer is needed to:
 - Determine the 1,048,576 beam paths and $1,048,576^2$ path distances
 - Solve 1,048,576 equations to determine the intensities, g_{ij} 's
 - Display the image using the 1,048,576 intensities

Reconstruction of image

Assume an image of $P = M \times N$ pixels, there are P beam equations where each beam traverses a subset of the P pixels.

As a result, the equation of beam path can have up to $P = M \times N$ terms.

$$\ln \frac{l_o^1}{l_i^1} = -(g_{11}d_{11}^1 + g_{12}d_{12}^1 + \dots + g_{1N}d_{1N}^1 + g_{21}d_{21}^1 + \dots + g_{2N}d_{2N}^1 + \dots + g_{M1}d_{1M}^1 + \dots + g_{MN}d_{NM}^1)$$

$$\ln \frac{l_o^2}{l_i^2} = -(g_{11}d_{11}^2 + g_{12}d_{12}^2 + \dots + g_{1N}d_{1N}^2 + g_{21}d_{21}^2 + \dots + g_{2N}d_{2N}^2 + \dots + g_{M1}d_{1M}^2 + \dots + g_{MN}d_{NM}^2)$$

⋮

$$\ln \frac{l_o^P}{l_i^P} = -(g_{11}d_{11}^P + g_{12}d_{12}^P + \dots + g_{1N}d_{1N}^P + g_{21}d_{21}^P + \dots + g_{2N}d_{2N}^P + \dots + g_{M1}d_{1M}^P + \dots + g_{MN}d_{NM}^P)$$

Define a vector of size $1 \times P$ for each of the beams :

$$l = [\ln \frac{l_o^1}{l_i^1}, \ln \frac{l_o^2}{l_i^2}, \dots, \ln \frac{l_o^P}{l_i^P}]$$

For the pixel intensity, define a matrix of size $M \times N$ and reshape the matrix to a vector of $1 \times P$:

$$g = [g_{11}, g_{21}, \dots, g_{N1}, g_{21}, \dots, g_{2N}, \dots, g_{M1}, \dots, g_{MN}]$$

For the distances for each beam, define a matrix d of size $P \times P$:

$$d = \begin{bmatrix} d_{11}^1, d_{12}^1, \dots, d_{1N}^1, d_{21}^1, \dots, d_{2N}^1, \dots, d_{1M}^1, \dots, d_{NM}^1 \\ d_{11}^2, d_{12}^2, \dots, d_{1N}^2, d_{21}^2, \dots, d_{2N}^2, \dots, d_{1M}^2, \dots, d_{NM}^2 \\ \vdots \\ d_{11}^P, d_{12}^P, \dots, d_{1N}^P, d_{21}^P, \dots, d_{2N}^P, \dots, d_{1M}^P, \dots, d_{NM}^P \end{bmatrix}$$

As a result, we have the set of $P = M \times N$ simultaneous equations:

$$l^T = dg^T$$

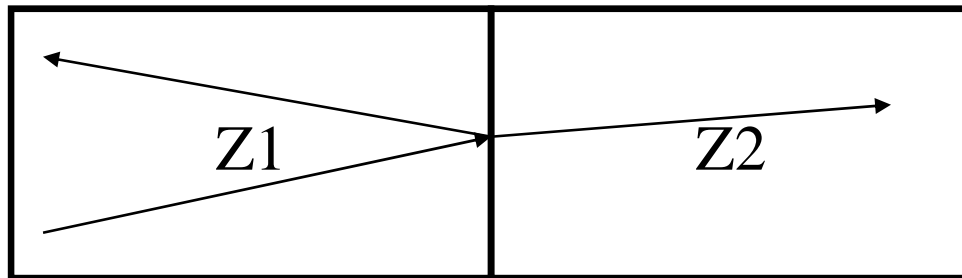
And we can solve for the intensities given l and d ,

$$g^T = d^l l^T$$

What Happens when we put Ultrasonic Energy into a Material?

- Attenuation in medium 1 due to the impedance Z_1
- Beam Splitting (scattering)
- Reflection at boundary

$$\Gamma = \left(\frac{Z_1 - Z_2}{Z_1 + Z_2} \right)^2$$



- For Air/Tissue Boundary
 - Most of the energy is reflected

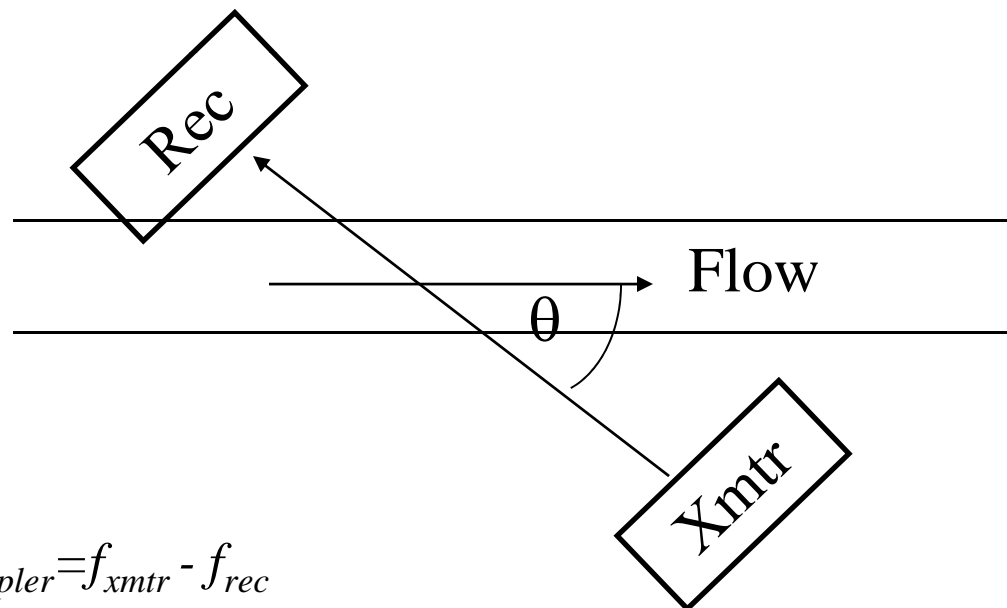
$$\Gamma = \left(\frac{50 - 150000}{50 + 150000} \right)^2 = .999$$

- For Jelly/Tissue Boundary
 - Most of the energy is transmitted

$$\Gamma = \left(\frac{148000 - 150000}{148000 + 150000} \right)^2 = .000045$$

Doppler Ultrasound

- Measure of blood flow in a blood vessel
- Doppler effect: change in frequency when object is moving relative to transmitter and receiver



- $f_{doppler} = f_{xmtr} - f_{rec}$
- $f_{doppler} = 2V \cos \theta$ V = velocity of blood; not the flow
- Need the cross-sectional area to calculate the flow; use ultrasound to perform this measurement

Magnetic Resonance Imaging

- MRI uses the principle of tomography
- Instead of Xrays, a magnetic field is used to interact with the magnetic properties of the body.
- The molecules and atoms of the body have spinning charged particles which cause a magnetic moment.
- When an external magnetic field is applied, these moments tend to align with the field.
- Most particles align N to S (low energy state) but could align N to N (high energy state).
- Any perturbation to cause a switch from the high energy state to the low state

Magnetic Resonance Imaging

- The energy state of the material at any time can be considered as the portion of the particles in the high energy state to the low energy state.
- In addition to spinning, the axes also precess or wobble (like a top) where the precessional (Larmor) frequency is $\omega = \gamma B$ where γ is called the gyromagnetic ratio and is a property of the particle and B is the applied magnetic field
- When energy is applied at the precessional frequency, resonance occurs and the particles become excited and move into the higher energy state.
- MRI measures the relaxation (energy change) back to the lower energy state.

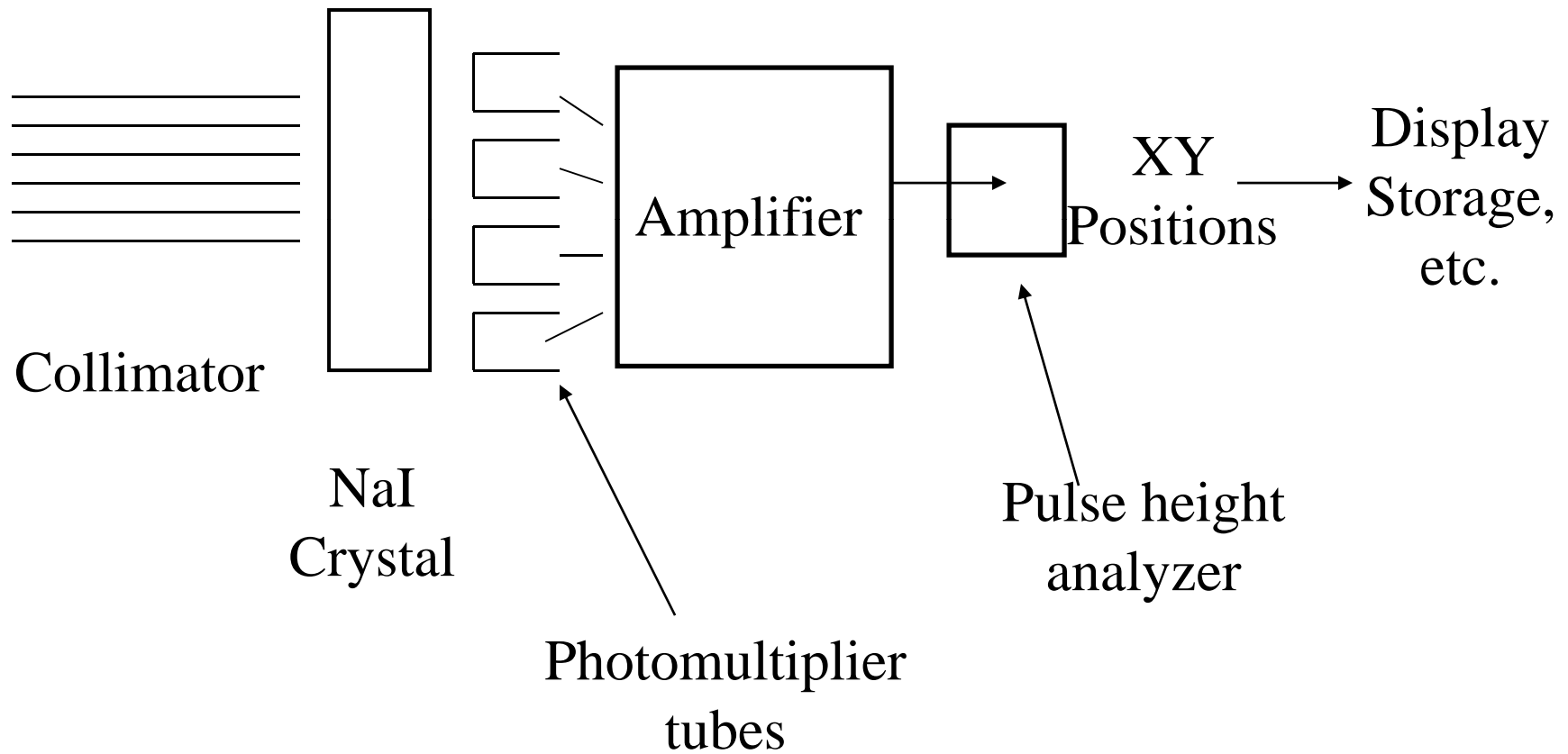
Functional MRI

- An MRI is taken of the brain activity while the subject is performing a function.
- This is used to determine the part of the brain which operates to handle the function being performed.
 - Odors
 - Visual
 - Thinking
 - Speech
 - Etc.

Nuclear Medicine

- Used for imaging organ function because a radioactive particles attach themselves to a particular organ
- Injection of a radioactive tracer and the resulting radioactivity is measured
- Radioactivity is detected with a sodium iodide crystal which produces blue light when hit with gamma radiation

Nuclear Medicine



Biological Effects (Hazards)

- No definitive information exists concerning safe levels
- No standards exist
- Factors which are important in safety are:
 - Frequency
 - Irradiation time
 - Beam intensity
 - Duty cycle
- Heat is Created

Analyzing the Image

- Signal Processing to determine the structures
 - Color Intensity
 - Contrast
 - Sharpness
 - Blur
 - Remove Noise
 - Etc.
- Mathematical Transformations
 - Fourier Transforms
 - 2D Fourier Transforms
 - Time Frequency Transformations
 - Filtering
 - Statistics

Analyzing the Image

- MATLAB Image Processing Toolkit
 - Preprocessing
 - Image Enhancement
 - Sharpening
 - Color Management
 - Image Transformations
 - Analysis
 - Statistics
 - Edge Detection
 - Noise Removal
 - Spatial Transformations
 - Also Displaying, importing and exporting images

Example



Original RGB
Image 465x393x3

```
imdata = imread('brainscan.jpg');  
figure,imshow(imdata);
```



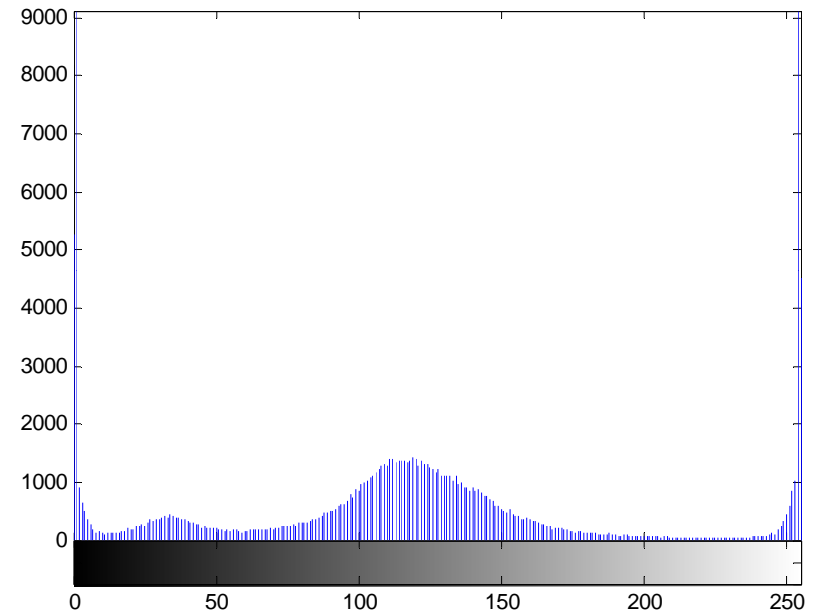
Grayscale
Image
465x393

```
J=rgb2gray(imdata);  
figure,imshow(J);
```

Example



Grayscale
Image
465x393



Histogram of
Grayscale
Image

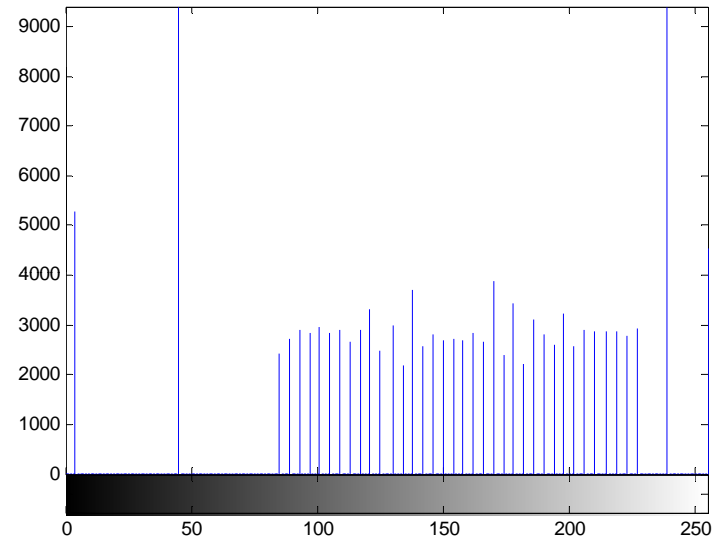
figure,imhist(J);

Example



Equalization of
Grayscale
Image

```
I2 = histeq(J);
```



Histogram of
Equalized
Grayscale
Image

Homework

1. A tomograph of the brain needs to be taken using 100 slices. The size of the image to be generated is 12 inches square. The finest detail to be imaged is 0.05 inch using a 3 pixel resolution.
 - i. Calculate the total number of pixels required to achieve the image resolution
 - ii. Assuming a 256 gray scale levels, calculate the size of the file needed to store this image.
 - iii. Assuming it takes 25 microseconds to take one measurement, calculate the total time to complete the full tomograph. Also assume that the calibration time is the same as the measurement time.
 - iv. Calculate long will it take to transmit the full tomograph over a dialup line (56Kb/s).

Homework

2. On the Website there are two text files:

Beam I/O which is the vector $l = [\ln \frac{l^1_o}{l^1_i}, \ln \frac{l^2_o}{l^2_i}, \dots, \ln \frac{l^P_o}{l^P_i}]$

Beam Distances which is the matrix

$$d = \begin{bmatrix} d^1_{11}, d^1_{12}, \dots, d^1_{1N}, d^1_{21}, \dots, d^1_{2N}, \dots, d^1_{1M}, \dots, d^1_{NM} \\ d^2_{11}, d^2_{12}, \dots, d^2_{1N}, d^2_{21}, \dots, d^2_{2N}, \dots, d^2_{1M}, \dots, d^2_{NM} \\ \vdots \\ d^P_{11}, d^P_{12}, \dots, d^P_{1N}, d^P_{21}, \dots, d^P_{2N}, \dots, d^P_{1M}, \dots, d^P_{NM} \end{bmatrix}$$

The original image (to generate these files) is 24 x 20 pixels and so the Beam I/O vector is 480 x 1 and the Beam Distances matrix is 480 x 480.

Find the original image. You should use MATLAB to help